

Numerical Method for Solving Obstacle Scattering Problems by an Algorithm Based on the Modified Rayleigh Conjecture

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Abstract. In this paper we present a numerical algorithm for solving the direct scattering problems by the Modified Rayleigh Conjecture Method (MRC) introduced in [1]. Some numerical examples are given. They show that the method is numerically efficient.

Key words. direct obstacle scattering problem, Modified Rayleigh Conjecture, MRC algorithm

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I. Introduction

The classical Rayleigh Conjecture is discussed in [4] and [5], where it is shown that, in general, this conjecture is incorrect: there are obstacles (for example, sufficiently elongated ellipsoids) for which the series, representing the scattered field outside a ball containing the obstacle, does not converge up to the boundary of this obstacle.

The Modified Rayleigh Conjecture (MRC) has been formulated and proved in [1] (see Theorem 1 below). A numerical method for solving obstacle scattering problems, based on MRC, was proposed in [1]. This method was implemented in [2] for two-dimensional obstacle scattering problems. The numerical results in [2] were quite encouraging: they show that the method is efficient, economical, and is quite competitive compared with the usual boundary integral equations method (BIEM). A recent paper [3] contains a numerical implementation of MRC in some three-dimensional obstacle scattering problems. Its results reconfirm the practical efficiency of the MRC method.

In this paper a numerical implementation of the Modified Rayleigh Conjecture (MRC) method for solving obstacle scattering problem in three-dimensional case is presented. Our aim is to consider more general than in [3] three-dimensional obstacles: non-convex, non-starshaped, non-smooth, and to study the performance of the MRC in these cases. The minimization problem (5) (see below), which is at the heart of the MRC method, is treated numerically in a new way, different from the one used in [2] and [3]. Our results present further numerical evidence of the practical efficiency of the MRC method for solving obstacle scattering problems.

The obstacle scattering problems (1)-(3), we are interested in, consists

of solving the equation

$$(\nabla^2 + k^2)u = 0 \quad \text{in } D' = R^3 \setminus D, \quad (1)$$

where $D \subset R^3$ is a bounded domain, satisfies the Dirichlet boundary condition

$$u|_S = 0, \quad (2)$$

where S is the boundary of D , which is assumed Lipschitz in this paper, and the radiation condition at infinity:

$$u = u_0 + v = u_0 + A(\alpha', \alpha) \frac{e^{ikr}}{r} + o\left(\frac{1}{r}\right) \quad r \rightarrow \infty, \quad (3)$$

$$r := |x|, \quad \alpha' = x/r, \quad u_0 := e^{ik\alpha \cdot x},$$

where v is the scattered field, $\alpha \in S^2$ is given, S^2 is the unit sphere in R^3 , $k = \text{const} > 0$ is fixed, k is the wave number. The coefficient $A(\alpha', \alpha)$ is called the scattering amplitude.

Denote

$$A_l(\alpha) := \int_{S^2} A(\alpha', \alpha) \overline{Y_l(\alpha')} d\alpha', \quad (4)$$

where $Y_l(\alpha)$ are the orthonormal spherical harmonics:

$$Y_l = Y_{lm}, \quad -l \leq m \leq l, \quad l = 0, 1, 2, \dots$$

$$Y_{lm}(\theta, \phi) = \frac{1}{\sqrt{2\pi}} e^{im\phi} \Theta_{lm}(\cos\theta),$$

$$\Theta_{lm}(x) = \sqrt{\frac{2l+1}{2} \frac{(l-m)!}{(l+m)!}} P_l^m(x),$$

$P_l^m(x)$ are the associated Legendre functions of the first kind,

$$P_l^m(x) := (1-x^2)^{m/2} \frac{d^m P_l(x)}{dx^m}, \quad m \geq 0,$$

and

$$P_l(x) := \frac{(-1)^l}{2^l l!} \frac{d^l}{dx^l} (1-x^2)^l.$$

For $m < 0$

$$\Theta_{lm}(x) = (-1)^m \Theta_{l,-m}(x).$$

Let $h_l(r)$ be the spherical Hankel functions of the first kind, normalized so that $h_l(kr) \sim e^{ikr}/r$ as $r \rightarrow +\infty$. Let $B_R := \{x : |x| \leq R\} \supset D$, and the origin is inside D .

Then in the region $r > R$, the solution to the acoustic wave problem (1)-(3) is of the form:

$$u(x, \alpha) = e^{ik\alpha \cdot x} + \sum_{l=0}^{\infty} A_l(\alpha) \psi_l(x), \quad |x| > R,$$

$$\psi_l := Y_l(\alpha') h_l(kr), \quad r > R, \quad \alpha' = x/r,$$

where

$$\sum_{l=0}^{\infty} := \sum_{l=0}^{\infty} \sum_{m=-l}^l.$$

Fix $\epsilon > 0$, an arbitrary small number. The following Lemmas and Theorem 1 are proved in [1].

Lemma 1. *There exist $L = L(\epsilon)$ and numbers $c_l = c_l(\epsilon)$ such that*

$$\|u_0(s) + \sum_{l=0}^L c_l(\epsilon) \psi_l(s)\|_{L^2(S)} < \epsilon. \quad (5)$$

Lemma 2. *If (5) holds, then $\|v_\epsilon(x) - v(x)\| = O(\epsilon)$, $\forall x \in D'$, $\epsilon \rightarrow 0$.*

where

$$v_\epsilon(x) := \sum_{l=0}^L c_l(\epsilon) \psi_l(x), \quad x \in D', \quad (6)$$

and

$$\|\cdot\| := \|\cdot\|_{H_{loc}^m(D')} + \|\cdot\|_{L^2(D'; (1+|x|)^{-\gamma})}, \quad \gamma > 0, m > 0, \quad (7)$$

m is arbitrary, and H^m is the Sobolev space.

Lemma 3. $c_l(\epsilon) \rightarrow A_l(\alpha), \forall l, \epsilon \rightarrow 0$.

Theorem 1 (Modified Rayleigh Conjecture). *Let $D \in R^3$ be a bounded obstacle with Lipschitz boundary S . For any $\epsilon > 0$ there exists $L = L(\epsilon)$ and $c_l(\epsilon) = c_{lm}(\epsilon)$, $0 \leq l \leq L$, $-l \leq m \leq l$, such that inequality (5) holds. If (5) holds then function (6) satisfies the estimate $\|v(x) - v_\epsilon(x)\| = O(\epsilon)$, where the norm is defined in (7). Thus, $v_\epsilon(x)$ is an approximation of the scattered field everywhere in D' .*

In order to obtain an accurate solution, usually one has to take L large. But as L grows the condition number of the matrix $(\psi_l, \psi_{l'})_{L^2(S)}$ is increasing very fast. So we choose some interior points $x_j \in D$, $j = 1, 2, \dots, J$, and use the following version of Theorem 1([2]):

Theorem 2. Suppose $x_j \in D$, $j = 1, 2, \dots, J$, then $\forall \epsilon > 0$, $\exists L = L(\epsilon)$ and $c_{lj}(\epsilon)$, $l = 0, \dots, L$, $j = 0, \dots, J(\epsilon)$, such that

(i)

$$\|u_0(s) + \sum_{j=0}^J \sum_{l=0}^L c_{lj}(\epsilon) \psi_l(s - x_j)\|_{L^2(S)} < \epsilon. \quad (5')$$

(ii)

$$\|v_\epsilon(x) - v(x)\| = O(\epsilon),$$

where

$$v_\epsilon(x) = \sum_{j=0}^J \sum_{l=0}^L c_{lj}(\epsilon) \psi_l(s - x_j)$$

and the $\|\cdot\|$ is defined in Lemma 2.

Remark. Theorem 1 is the basis for MRC algorithm for computation of the field scattered by an obstacle: one takes an $\epsilon > 0$ and an integer $L > 0$, minimizes the left-hand side of (5) with respect to c_l , and if the minimum is $\leq \epsilon$ then the function (6) is the approximate solution of the obstacle scattering problem with the accuracy $O(\epsilon)$. If the above minimum is greater than ϵ , then one increases L until the minimum is less than ϵ . This is possible by Lemma 1. In computational practice, one may increase also the number J of points x_j inside D , as explained in Theorem 2. The increase of J allows one to reach the desired value of the above minimum keeping L relatively small. This gives computational advantage in many

cases.

In section 2, an algorithm is presented for solving the problem (1)-(3). This algorithm is based on the MRC. Compared with the previous work in the case of two- and three-dimensional MRC([2],[3]), we consider more general surfaces, in particular non-starshaped and piecewise-smooth boundaries. The numerical results are given in section 3. A discussion of the numerical results is given in section 4.

II. The MRC algorithm for Solving Obstacle Scattering Problems

1. Smooth starshaped boundary:

Assume the surface S is given by the equation

$$r = r(\theta, \varphi), \quad 0 \leq \varphi \leq 2\pi, \quad 0 \leq \theta \leq \pi.$$

Define

$$F(c_0, c_1, \dots, c_L) := \|u_0 + \sum_{l=0}^L c_l \psi_l\|_{L^2(S)}^2. \quad (5'')$$

Let

$$h_1 = 2\pi/n_1, \quad h_2 = \pi/n_2$$

$$0 = \varphi_0 < \varphi_1 < \dots < \varphi_{n_1} = 2\pi, \quad \varphi_{i_1} = i_1 h_1, \quad i_1 = 1, \dots, n_1,$$

$$0 = \theta_0 < \theta_1 < \dots < \theta_{n_2} = \pi, \quad \theta_{i_2} = i_2 h_2, \quad i_2 = 1, \dots, n_2,$$

where n_1 and n_2 are the number of steps. By Simpson's formula([8]), we obtain an approximation of $F(c_0, c_1, \dots, c_L)$:

$$F(c_0, c_1, \dots, c_L) = \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} a_{i_1 i_2} |u_{0 i_1 i_2}| + \sum_{l=0}^L c_l |\psi_{l i_1 i_2}|^2 w_{i_1 i_2} h_1 h_2 \quad (5''')$$

where

$$a_{i_1, i_2} = \begin{cases} 4, & i_1 \text{ and } i_2 \text{ even} \\ 8, & i_1 - i_2 \text{ odd} \\ 16, & i_1 \text{ and } i_2 \text{ odd} \end{cases}$$

and

$$\psi_{l i_1 i_2} = Y_l(\theta_{i_1}, \varphi_{i_2}) h_l(kr(\theta_{i_1}, \varphi_{i_2})), \quad w_{i_1 i_2} = w(\theta_{i_1}, \varphi_{i_2})$$

where

$$w(\theta, \varphi) = (r^2 r_\varphi^2 + r^2 r_\theta^2 \sin^2 \theta + r^4 \sin^2 \theta)^{1/2}. \quad (8)$$

We can find $c^* = (c_0^*, c_1^*, \dots, c_L^*)$ such that

$$F(c^*) = \min F(c_0, c_1, \dots, c_L). \quad (9)$$

We first write

$$F(c) = \|Ac - B\|^2, \quad (10)$$

where

$$A = (A_{l,i})_{M \times L_1}, \quad A_{l,i} = \psi_{l i_1 i_2} (a_{i_1 i_2} w_{i_1 i_2} h_1 h_2)^{\frac{1}{2}}, \quad i = i_1 i_2,$$

$$B = (B_i)_{M \times 1}, \quad B_i = u_{0i_1i_2} (a_{i_1i_2} w_{i_1i_2} h_1 h_2)^{\frac{1}{2}},$$

in which $M = n_1 n_2$, $L_1 = (L + 1)(2L + 1)$ since $c_l = c_{lm}$, $0 \leq l \leq L$, $-l \leq m \leq l$.

Then Householder reflections are used to compute an orthogonal-triangular factorization: $A * P = Q * R$ where P is a permutation([8], p.171), Q is an orthogonal matrix, and R is an upper triangular matrix. Let $r = \text{rank}(A)$. This algorithm requires $4ML_1r - 2r^2(M + L_1) + 4r^3/3$ flops([9], pp.248-250). The least squares solution c is computed by the formula $c = P * (R^{-1} * (Q' * (A^T B)))$. This minimization procedure is based on the matlab code([10]).

In [2] and [3] singular value decomposition was used for minimization of (5"). Here we use the matlab minimization code which is based on a factorization of the matrix A . This has the following advantages from the point of view of numerical analysis. We can choose an integer r_1 :

$$0 < r_1 \leq r$$

such that the first r_1 rows and columns of R form a well-conditioned matrix when A is not of full rank, or the rank of A is in doubt([10]). See Golub and Van Loan [9] for a further discussion of numerical rank determination.

If we choose $x_j \in D$, $j = 1, \dots, J$, we obtain

$$\begin{aligned} F_J(c) &= F_J(c_{01}, \dots, c_{0J}, c_{11}, \dots, c_{1J}, \dots, c_{L1}, \dots, c_{LJ}) \\ &= \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \sum_{j=1}^J a_{i_1 i_2} |u_{0 i_1 i_2}| + \sum_{l=0}^L c_{lj} |\psi_{l i_1 i_2}|^2 w_{i_1 i_2} h_1 h_2. \end{aligned}$$

The algorithm for finding the minimum of $F_J(c)$ will be same.

2. Piecewise-smooth boundary:

Suppose

$$S = \bigcup_{n=1}^N S_n.$$

Then

$$F(c_0, c_1, \dots, c_L) = \sum_{n=1}^N \|u_0 + \sum_{l=0}^L c_l \psi_l\|_{L^2(S_n)}^2$$

$$\forall (x, y, z) \in S_n, \quad r^2 = x^2 + y^2 + z^2, \quad \cos \theta = z/r, \quad \tan \varphi = y/x. \quad (11)$$

3. Non-starshaped case:

Suppose S is a finite union of the surfaces, each of which is starshaped with respect to a point \vec{r}_n^0 ,

$$S = \bigcup_{n=1}^N S_n.$$

and the the surfaces S_n are given by the equations in local spherical coordinates:

$$S_n : \quad \vec{r} - \vec{r}_n^0 = (r_n(\theta_n, \varphi_n) \cos \varphi_n \sin \theta_n, r_n(\theta_n, \varphi_n) \sin \varphi_n \sin \theta_n, r_n(\theta_n, \varphi_n) \cos \theta_n),$$

$$n = 1, \dots, N,$$

where \vec{r}_n^0 are constant vectors.

Then

$$F(c_0, c_1, \dots, c_L) = \sum_{n=1}^N \|u_0 + \sum_{l=0}^L c_l \psi_l\|_{L^2(S_n)}^2.$$

The weight functions $w_n(\theta, \varphi)$ are the same as in (8) since \vec{r}_n^0 are constant vectors.

III. Numerical Results

In this section, we give four examples to show the convergence rate of the algorithm and how the error depends on the shape of S .

Example 1. The boundary S is the sphere of radius 1 centered at the origin.

In this example, the exact coefficients are:

$$c_{lm} = -\frac{4\pi i^l j_l(k)}{h_l(k)} \overline{Y_{lm}(\alpha)}$$

Let $k = 1$, $\alpha = (1, 0, 0)$. We choose $n_1 = 20$, $n_2 = 10$.

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
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$F(c^*)$	6.3219	1.6547	0.2785	0.0368	0.0034	0.0003	0.0000	0.0000
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err(c)	0.0303	0.0172	0.0020	0.0004	0.0000	0.0000	0.0000	0.0000
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where

$$err(c) = \left(\sum_{l=0}^L |c_l^* - c_l|^2 \right)^{\frac{1}{2}}.$$

When $n_1 = 40$, $n_2 = 20$,

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
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$F(c^*)$	6.3544	1.6562	0.2820	0.0358	0.0036	0.0003	0.0000	0.0000
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$err(c)$	0.0147	0.0076	0.0011	0.0001	0.0000	0.0000	0.0000	0.0000
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Next, we fix $n_1 = 20$, $n_2 = 10$ and test the results for different k and α .

When $k = 2$, $\alpha = (1, 0, 0)$,

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
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$F(c^*)$	10.4506	5.5783	1.9291	0.5217	0.0970	0.0156	0.0020	0.0003
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$err(c)$	0.0404	0.0205	0.0048	0.0020	0.0005	0.0000	0.0000	0.0000
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When $k = 1$, $\alpha = (0, 1, 0)$,

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
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$F(c^*)$	6.3801	1.6628	0.2821	0.0371	0.0044	0.0003	0.0000	0.0000
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$\text{err}(c)$	0.0014	0.0106	0.0005	0.0004	0.0000	0.0000	0.0000	0.0000
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When $k = 1$, $\alpha = (0, 0, 1)$,

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
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$F(c^*)$	6.4156	1.6909	0.2955	0.0418	0.0025	0.0002	0.0000	0.0000
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$\text{err}(c)$	0.0093	0.0109	0.0049	0.0007	0.0001	0.0000	0.0000	0.0000
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When $k = 1$, $\alpha = (1/\sqrt{2}, 1/\sqrt{2}, 0)$,

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
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$F(c^*)$	6.3500	1.6711	0.2810	0.0371	0.0040	0.0003	0.0000	0.0000
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$\text{err}(c)$	0.0218	0.0057	0.0019	0.0004	0.0001	0.0000	0.0000	0.0000
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When $k = 1$, $\alpha = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$,

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
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$F(c^*)$	6.3739	1.6542	0.2850	0.0368	0.0040	0.0003	0.0000	0.0000
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$\text{err}(c)$	0.0170	0.0054	0.0021	0.0003	0.0001	0.0000	0.0000	0.0000
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Example 2. The boundary S is the surface of the cube $[-1, 1]^3$. Here

$$S = \bigcup_{n=1}^6 S_n.$$

and

$$\begin{aligned} F(c_0, c_1, \dots, c_L) &= \sum_{n=1}^6 \|u_0 + \sum_{l=0}^L c_l \psi_l\|_{L^2(S_n)}^2 \\ &= \sum_{n=1}^6 \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} a_{i_1 i_2} |u_{0 i_1 i_2} + \sum_{l=0}^L c_l \psi_{l i_1 i_2}|^2 \Delta_1 \Delta_2 \end{aligned}$$

where

$$\Delta_1 = 2/n_1, \quad \Delta_2 = 2/n_2.$$

The origin is chosen at the center of symmetry of the cube. The surface area element is calculated in the Cartesian coordinates, so the weight $w = 1$.

Let S_1 be the surface

$$z = 1, \quad -1 \leq x \leq 1, \quad -1 \leq y \leq 1$$

and

$$x_{i_1} = -1 + i_1 \Delta_1, \quad 0 \leq i_1 \leq n_1$$

$$y_{i_2} = -1 + i_2 \Delta_2, \quad 0 \leq i_2 \leq n_2$$

Then

$$\psi_{l_{i_1 i_2}} = Y_l(\theta_{i_1}, \varphi_{i_2}) h_l(kr(\theta_{i_1}, \varphi_{i_2})),$$

and θ_{i_1} and φ_{i_2} can be computed by formula (11). For other surfaces S_j the algorithm is similar.

The values of $\min F(c) = F(c^*)$ and the values $\min F_J(c) = F_J(c^*)$ with x_j :

$$\{x_j : j = 0, \dots, 6\} = \{(0, 0, 0), (0.2, 0, 0), (-0.2, 0, 0),$$

$$(0, 0.2, 0), (0, -0.2, 0), (0, 0, 0.2), (0, 0, -0.2)\}$$

are given below.

We choose $n_1 = 10, n_2 = 10$

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000	8.0000
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$F(c^*)$	10.6301	3.6277	2.6760	2.2309	1.9832	1.5737	1.5034	1.2948	1.1753
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$F_J(c^*)$	2.6297	1.0970	0.5487	0.1572	0.0667	0.0320	0.0168	0.0078	0.0035
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When $n_1 = 20$, $n_2 = 20$,

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000	8.0000
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$F(c^*)$	10.7923	3.7144	2.7778	2.3393	2.0873	1.6671	1.5938	1.4277	1.3368
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$F_J(c^*)$	2.7248	1.1433	0.5757	0.1686	0.0694	0.0652	0.0236	0.0143	0.0090
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Example 3. The boundary S is the surface of the ellipsoid $x^2 + y^2 + z^2/b^2 = 1$, the values of $\min F(c) = F(c^*)$, $b = 2, 3, 4, 5$ with $n_1 = 20$, $n_2 = 10$ are:

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
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b=2	8.8836	5.4955	3.0421	2.8434	1.3622	1.2093	0.8753	0.8132
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b=3	14.1617	12.0477	7.2296	7.0999	3.8077	3.6829	3.1324	3.0496
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b=4	19.5326	17.9346	9.9927	9.8720	5.3333	5.2008	4.6793	4.5738
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b=5	22.9765	21.5653	11.4850	11.3587	6.1637	6.0096	5.5202	5.3933
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The values of $\min F_J(c) = F_J(c^*)$, $b = 2, 3, 4, 5$ with x_j :

$$\{x_j : j = 0, \dots, 6\} = \{(0, 0, 0), (0.5, 0, 0), (-0.5, 0, 0),$$

$$(0, 0.5, 0), (0, -0.5, 0), (0, 0, 0.5), (0, 0, -0.5)\}$$

are:

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
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b=2	2.4856	0.7090	0.2530	0.0062	0.0000	0.0000	0.0000	0.0000
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b=3	4.6639	1.3619	0.6618	0.0074	0.0000	0.0000	0.0000	0.0000
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b=4	5.5183	1.8624	0.7844	0.0060	0.0000	0.0000	0.0000	0.0000
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b=5	11.0579	8.7027	6.4831	0.8357	0.0017	0.0000	0.0000	0.0000
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Example 4. The obstacle is a dumbbell. Its boundary S is not smooth, non-starshaped and not convex:

$$S = S_1 \cup S_2 \cup S_3$$

$$S_1 : \vec{r} - (0, 0, 1) = (1.5 \cos \varphi \sin \theta, 1.5 \sin \varphi \sin \theta, 1.5 \cos \theta)$$

$$S_2 : \vec{r} - (0, 0, -1) = (1.5 \cos \varphi \sin \theta, 1.5 \sin \varphi \sin \theta, 1.5 \cos \theta)$$

$$S_3 : r \sin \theta = 1$$

$$\{x_j : j = 0, \dots, 10\} = \{(0, 0, 0), (0, 0, 0.1), (0, 0, -0.1), (0, 0, 0.2), (0, 0, -0.2),$$

$$(0, 0, 0.3), (0, 0, -0.3), (0, 0, 0.4), (0, 0, -0.4), (0, 0, 0.5), (0, 0, -0.5)\};$$

We choose $n_1 = 20$, $n_2 = 10$ for every $S_i (i = 1, 2, 3)$.

L	0	1.0000	2.0000	3.0000	4.0000	5.0000	6.0000	7.0000
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$F(c^*)$	25.8840	20.8059	16.4968	15.6622	12.9241	12.1915	11.0187	9.5263
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$F_J(c^*)$	20.3118	8.0238	5.1062	2.5908	0.8304	0.4067	0.0453	0.0084
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IV. Conclusion

From the numerical results one can see that the accuracy of the numerical solution depends on the smoothness and elongation of the object.

In Example 1 the surface S is a unit sphere and the numerical solution is very accurate. In Example 3 the results for different elongated ellipsoids show that if the elongation (eccentricity) grows, then the accuracy decreases. In Example 2 the surface is not smooth and the result is less accurate than in Example 3. In Example 4 the surface is nonconvex and not smooth, but the accuracy is of the same order as in Example 2.

When b is large or S is not smooth, the numerical results in Example 2 and Example 3 show that if one adds more points x_j then the accuracy of the solution increases.

In Example 1 and Example 2, as one increased n_1 and n_2 , the minimum

$F(c^*)$ has also increased because the condition number of the matrix A in (10) grew as n_1 and n_2 increased.

Using the results of Example 1 one can check the accuracy in finding c_l by the value of the minimum

$$F(c^*) \leq \epsilon.$$

References

- [1] Ramm A. G. [2002], Modified Rayleigh Conjecture and Applications, J. Phys. A: Math. Gen. 35, L357-L361.
- [2] Gutman S. and Ramm A. G. [2002], Numerical Implementation of the MRC Method for Obstacle Scattering Problems, J. Phys. A: Math. Gen. 35, L8065-L8074.
- [3] Gutman S. and Ramm A. G., Modified Rayleigh Conjecture Method for Multidimensional Obstacle Scattering Problems(submitted).
- [4] Barantsev, R., Concerning the Rayleigh hypothesis in the problem of scattering from finite bodies of arbitrary shapes, Vestnik Lenigrad. Univ., Math., Mech., Astron., 7, (1971), 52-62.
- [5] Millar, R., The Rayleigh hypothesis and a related least-squares solution to scattering problems for periodic surfaces and other scatterers, Radio, Sci., 8, (1973), 785-796.
- [6] Ramm, A. G., Scattering by obstacles, D. Reidel, 1986.
- [7] Triebel H., Theory of Function Spaces, vol. 78 of Monographs in Mathematics. Birkhauser Verlag, Basel, 1983.
- [8] Kincaid D. and Cheney W., Numerical Analysis: Mathematics of Scientific Computing, Brooks/Cole, 2002.
- [9] Golub G. H. and Van Loan C. F., Matrix Computations, The John

Hopkins University Press: Baltimore and London, 1996.

[10] Anderson, E., Z. Bai, C. Bischof, S. Blackford, J. Demmel, J. Don-
garra, J. Du Croz, A. Greenbaum, S. Hammarling, A. McKenney, and D.
Sorensen, LAPACK User's Guide (http://www.netlib.org/lapack/lug/lapack_lug.html),
Third Edition, SIAM, Philadelphia, 1999.